

D1.1. Mathematical framework to compare efficiency of non-sampling based methods and alternative power flow formulations

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This deliverable is based on the publication:

Yurtseven, K., Koirala, A., Ergun, H., Van Hertem, D. with Yurtseven, K. (corresp. author) (2025). Stochastic optimal power flow for hybrid AC/DC grids considering continuous non-Gaussian uncertainty. *International Journal Of Electrical Power & Energy Systems*, 170, Art.No. 110828. [doi: 10.1016/j.ijepes.2025.110828](https://doi.org/10.1016/j.ijepes.2025.110828)



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1. Purpose within the project

This paper forms the base stochastic optimization layer of the project. It develops a chance constrained stochastic optimal power flow model for hybrid AC/DC grids in which renewable generation uncertainty is represented in a continuous non-Gaussian space, HVDC converter set-points are optimized, and renewable curtailment is treated as an endogenous decision variable. The paper is therefore not only a methodological contribution; it defines the operating problem that the later security-constrained, optimal switching, and risk-based studies build upon.

High renewable penetration creates bounded and often asymmetric forecast errors, while hybrid AC/DC grids introduce controllable power flow paths that can reduce congestion. The proposed formulation captures these elements in one model. Generator dispatch, converter operation, bus voltages, branch flows, and renewable curtailment are all represented by Polynomial Chaos Expansion coefficients, so the optimization returns both expected operating decisions and probability distributions of the quantities of interest.

Aspect	Summary
Main method	Intrusive Polynomial Chaos Expansion with Galerkin projection and moment-based chance constraints.
Grid representation	Hybrid AC/DC optimal power flow in current-voltage variables, including AC branches, DC branches, converters, converter losses, and RES curtailment.
Uncertainty	Gaussian load uncertainty and bounded non-Gaussian RES forecast errors, represented with polynomial bases matched to the input distributions.
Main outputs	Expected generation cost, RES curtailment distributions, converter set-point distributions, branch flow distributions, and scalability indicators.

2. Methodological contribution

The methodological contribution is the integration of three modeling requirements that had not been combined in a single SOPF framework: continuous non-Gaussian uncertainty, HVDC compatibility, and RES curtailment as a decision variable. Polynomial Chaos Expansion is used to represent each random input as a linear combination of orthogonal polynomials. Equality constraints are projected into deterministic coefficient space using Galerkin projection. Inequality constraints are reformulated as chance constraints using the first two moments, which can be computed directly from the PCE coefficients. This eliminates repeated scenario sampling while preserving a continuous uncertainty representation.

Figure 1 summarizes the workflow. The left part of the figure transforms forecast error distributions and grid data into stochastic grid data. The central part reformulates the deterministic power flow model into a coefficient based stochastic optimization model. The right part is essential for the project final report because it shows that the method produces operational probability distributions, not only point estimates. This makes the framework suitable for evaluating the risk of curtailment, branch loading, converter loading, and generation dispatch.

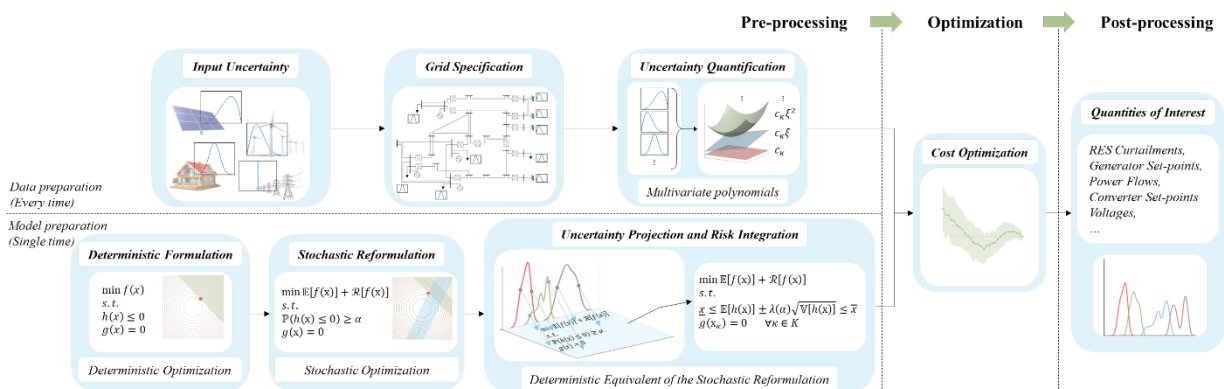


Figure 1: Workflow of the PCE-based Stochastic Optimal Power Flow framework

3. Case Study Results

Case 1 validates the PCE-based optimization against Monte Carlo simulation on a 5-bus hybrid AC/DC test system. The system includes five AC buses, five generators, six AC branches, three DC buses, three HVDC branches, and converters. Load uncertainty follows a Gaussian distribution and RES uncertainty follows a Beta distribution. The Monte Carlo convergence check confirms that 2000 iterations are sufficient, and the output distributions obtained from PCE closely match those obtained from Monte Carlo. The maximum deviation in expected operational cost across penetration levels is approximately 1.5%. The computational comparison is particularly important for the project: 500 Monte Carlo iterations require approximately 121 seconds for one penetration level, while the PCE formulation obtains the corresponding result in under 2 seconds. This demonstrates why the project framework can be used as an optimization tool rather than only as an offline uncertainty propagation method.

Case 2 evaluates the value of HVDC flexibility by comparing a purely AC grid and a hybrid AC/DC grid with equivalent line capacities. Figure 2 shows the result at high RES penetration levels. The hybrid AC/DC grid consistently yields lower expected RES curtailment and narrower curtailment distributions. The hosting capacity result is also central: the purely AC grid reaches its curtailment-free hosting capacity at about 85% RES penetration, whereas the hybrid AC/DC grid can accommodate penetration levels up to about 105% without RES curtailment for grid security. At penetration levels where curtailment is not required, the expected hourly operation cost difference is around 50% in favor of the hybrid AC/DC grid, as illustrated in Figure 3. The figure therefore supports a clear project conclusion: HVDC controllability increases the usable flexibility of the grid under uncertainty, and this effect is visible both in expected costs and in the full curtailment distribution.

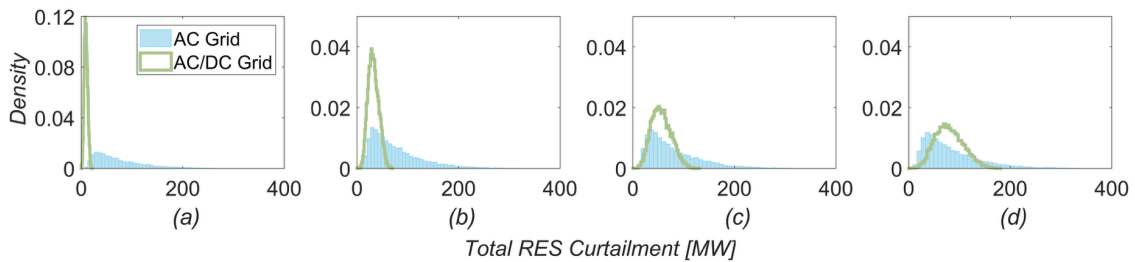


Figure 2: RES curtailment distributions in AC and hybrid AC/DC grids at 110% (a), 115% (b), 120% (c), and 125% (d) RES penetration.

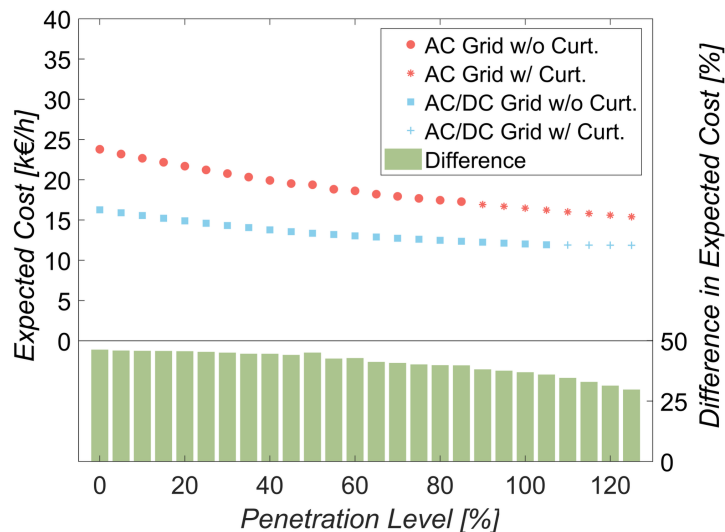


Figure 3: Simulation results of the proposed PCE-based SOPF for hybrid AC/DC and AC grids

Case 3 studies the consequences of simplifying RES forecast errors as Gaussian. The 67-bus hybrid AC/DC system includes 67 AC buses, 17 synchronous generators, 3 wind farms, 102 AC branches, 9 DC buses, converters, and 11 HVDC branches. Figure 4 compares curtailment distributions under Gaussian and Beta assumptions at confidence levels from 99% to 85%. The results show that the Gaussian assumption can shift both expected curtailment and the high curtailment region of the distribution. The numerical consequences are large. Compared with the Beta case, the Gaussian case implies potential annual expected cost mismatches of approximately EUR 4.42 million at 99% confidence, EUR 1.02 million at 95%, EUR 1.37 million at 90%, and EUR 455 thousand at 85%. The expected total curtailment mismatch reaches 73.84 MW, 12.50 MW, 15.02 MW, and 26.22 MW across the same confidence levels. The 0.99-quantile mismatch in total curtailment reaches 7.23 MW, 97.85 MW, 170.32 MW, and 186.77 MW, respectively. These results justify the use of non-Gaussian uncertainty in the project because the modeling choice changes economically relevant decisions.

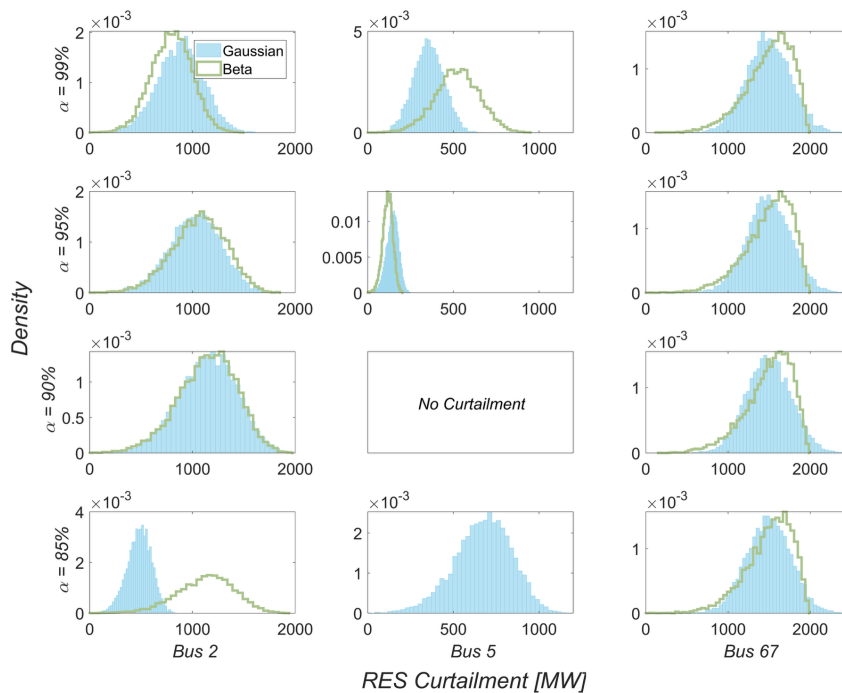


Figure 4: RES curtailment distributions under Gaussian and Beta input assumptions

Case 4 provides the scalability evidence that was missing from the shorter report. The test systems include the 5-bus and 67-bus systems used earlier, a modified hybrid AC/DC IEEE 118-bus system, and a 588-bus hybrid AC/DC system. The 118-bus case contains 118 AC buses, 54 synchronous generators, 20 wind farms, 186 AC branches, 4 DC buses, 4 converters, and 4 HVDC branches. The 588-bus case contains 588 AC buses, 167 synchronous generators, 40 wind farms, 686 AC branches, 7 DC buses, 7 converters, and 8 HVDC branches. Simulations are performed at 95%, 90%, and 85% confidence levels and at 0%, 30%, 60%, and 90% RES penetration. The quartile ranges of computational time are 1.17 to 1.87 seconds for the 5-bus system, 58.5 to 71.5 seconds for the 67-bus system, 76.5 to 181 seconds for the 118-bus system, and 624.8 to 868.8 seconds for the 588-bus system, as depicted in Figure 5. The largest reported average time is 1112.15 seconds for the 588-bus case with the higher mean RES uncertainty setting at 60% penetration and 85% confidence. The scalability results show that the method remains tractable even for large hybrid AC/DC grids, although higher expected RES injections and tighter chance constraints can increase solution time.

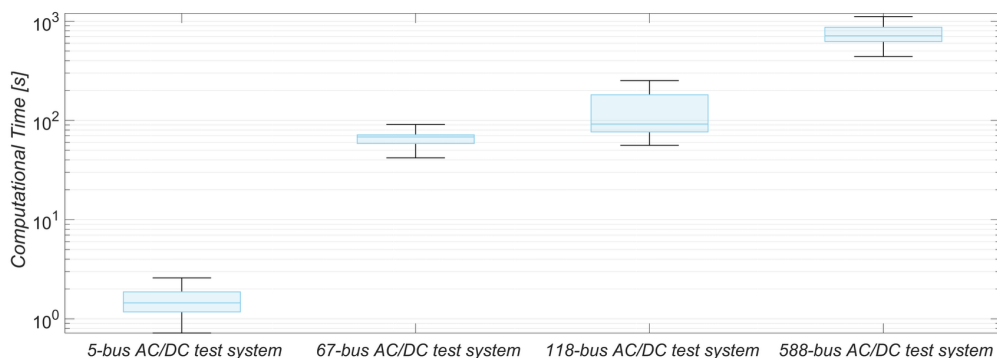


Figure 5: Computational times of the proposed SOPF framework with different system sizes under various confidence levels, penetration levels, and uncertainty characteristics.

4. Expanded interpretation of the complete result set

Taken together, the four case studies establish a complete validation chain for the project. The first case study demonstrates numerical accuracy and speed relative to Monte Carlo simulation. The second converts this mathematical accuracy into an operational conclusion by showing that controllable HVDC links reduce curtailment and increase hosting capacity. The third demonstrates why the uncertainty model matters by showing that the Gaussian simplification can alter both expected curtailment and high quantile curtailment values. The fourth confirms that the method is not restricted to small illustrative networks. This sequence is important because the project objective is not only to formulate a stochastic OPF model, but to provide a computationally credible congestion management method that can be extended to more realistic systems.

The scalability study deserves particular emphasis in the final report because it changes the interpretation of PCE from an accurate but potentially expensive uncertainty quantification method into a practically usable optimization approach. The model complexity grows with the number of independent stochastic germs rather than with the number of uncertain injections. This distinction is central for transmission applications, where many RES units may share a smaller number of forecast error drivers. The 588-bus experiments show that solving a continuous non-Gaussian chance-constrained hybrid AC/DC OPF is computationally feasible when this structure is exploited. The results also show that solution time depends not only on network size, but also on the uncertainty level, RES penetration, and confidence level. For example, higher expected RES availability may increase the need to manage branch and voltage constraints probabilistically, which explains why some high mean cases are more computationally demanding.

The Gaussian versus Beta comparison also has a direct operational implication. If a Gaussian approximation is used near the physical bounds of renewable generation, the model may assign probability to infeasible production levels and distort the shape of the resulting curtailment distribution. The reported cost mismatches and quantile mismatches are therefore not cosmetic differences in fitted probability curves. They translate into different curtailment expectations, different high-risk curtailment levels, and ultimately different congestion management decisions. For a project that aims to support uncertainty-aware operation, this result justifies the use of distribution-specific polynomial bases rather than generic Gaussian assumptions.

The paper also clarifies the economic role of RES curtailment as a decision variable. Curtailment is often treated as an undesirable residual outcome after other grid constraints are enforced. In this formulation, curtailment becomes a controlled flexibility source whose distribution is optimized jointly with conventional generation and HVDC set-points. This is the key reason why the framework can later be extended toward risk-based objectives, security constraints, and balancing-aware redispatch. Without this endogenous curtailment representation, the later project layers would not have a meaningful control variable to reshape the operational cost distribution under uncertainty.