

D1.2 Methodology to represent discrete decision variables in probabilistic optimisation models

Authors: Kaan Yurtseven, Hakan Ergun

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1. Purpose within the project

This paper adds topology control to the project framework. The preceding models optimize continuous set-points such as generation, converter power, and curtailment. This work introduces stochastic optimal transmission switching, so the model can also choose the AC and DC branches that remain connected. Transmission switching is a non-costly remedial action, and its integration is directly aligned with the project objective of reducing the use of costly remedial actions such as redispatch and renewable curtailment.

The main methodological challenge is the combination of discrete switching variables with a continuous PCE uncertainty space. Scenario-based formulations typically replicate binary decisions across scenarios and then impose non-anticipativity constraints. This paper instead enforces non-anticipativity directly in the PCE expansion by setting the higher-order coefficients of switching variables to zero. The switching state is therefore fixed across uncertainty realizations, while generator dispatch, converter set-points, and curtailment can remain adaptive wait-and-see variables.

Aspect	Summary
Main method	Chance constrained stochastic optimal transmission switching for hybrid AC/DC grids using intrusive PCE.
New modeling layer	Discrete AC and DC branch switching decisions inside a continuous non-Gaussian uncertainty framework.
Tractability device	Binary relaxation combined with solver tolerance analysis, enabling solution with Ipopt instead of relying on full MINLP solution.
Validation scope	5-bus economic and computational comparison, 67-bus sensitivity analysis, 588-bus scalability analysis.

2. Methodological contribution

Figure 1 illustrates the time structure used in the formulation. The topology reconfiguration variable is a here-and-now decision made at the beginning of the stage. It cannot vary with a later forecast error realization. The generator dispatch, RES curtailment, and HVDC converter set-points are wait-and-see variables and can adapt to uncertainty. This distinction is essential. If the switching variable were allowed to vary across the continuous uncertainty space, the model would imply a different physical grid topology for different realizations inside the same operating interval, which is not a realistic operational decision.

The deterministic transmission switching equations are formulated in an IVR representation. Switching variables disconnect voltage relationships and enforce zero current flow on disconnected lines. The full mixed-integer nonlinear version is computationally demanding, so the proposed model relaxes the binary variables through complementarity-type constraints. Because relaxed binary variables can affect probability distributions even when the expected objective is similar, the paper does not only report cost. It also performs a sensitivity analysis of the binary relaxation parameter and solver tolerance, which is a practical contribution for using the method on large grids.

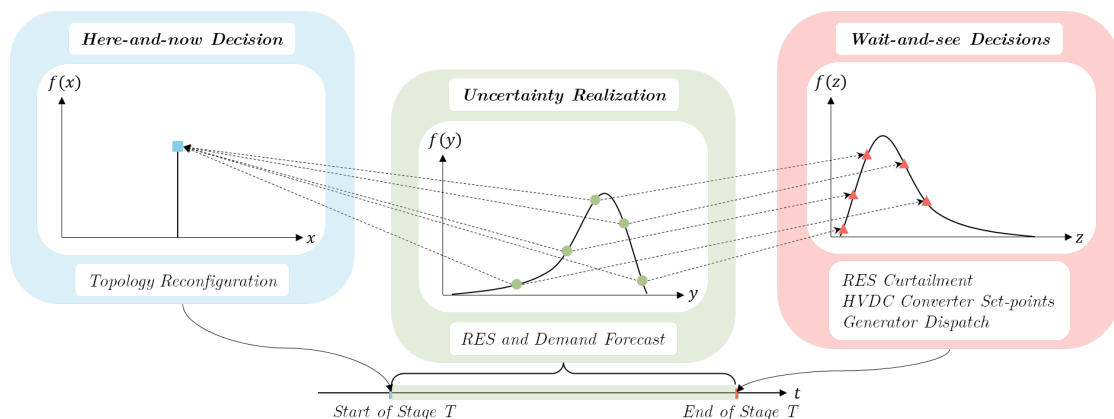


Figure 1: Here-and-now and wait-and-see decision structure of the SOTS model

3. Case study results

Case 1 compares three models on the 5-bus hybrid AC/DC system: SOPF without transmission switching, MINLP SOTS without binary relaxation, and the proposed relaxed SOTS model. The SOTS models determine an optimal topology that disconnects one AC branch and one DC branch. Figure 2 shows that both SOTS formulations consistently reduce expected operational costs compared with SOPF. Cost savings increase up to around 65% penetration, continue at a slower rate from 65% to 80%, and then diminish beyond 80% because local wind generation at all buses increasingly serves local demand. The relaxed SOTS objective remains very close to the MINLP SOTS objective. Differences are near zero up to 75% penetration and remain below 1% even when curtailment probability distributions become active at higher penetration.

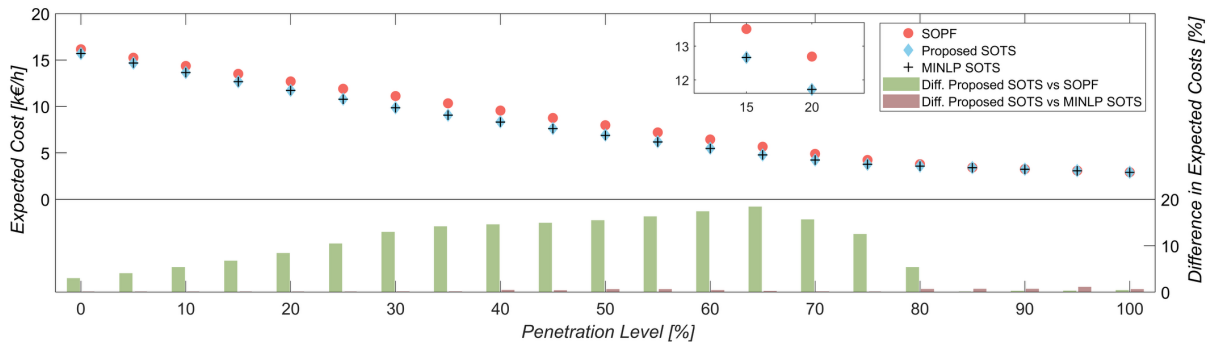


Figure 2: Expected operational cost comparison between SOPF, MINLP SOTS, and the proposed SOTS

The curtailment results in Figure 3 are central for the project final report. Transmission switching reduces both the expected value and the extreme values of total RES curtailment. Relative to the SOPF case, expected RES curtailment decreases by 18.92% at 75% penetration, 12.89% at 80% penetration, and 2.14% at 85% penetration. The smaller reduction at 85% does not invalidate the method; it indicates that as the system becomes more renewable dominated and local injections become large, the room for beneficial topology changes decreases. The result shows that topology control is most valuable in the operating range where congestion is significant but the system has enough flow diversity for switching to redirect power.

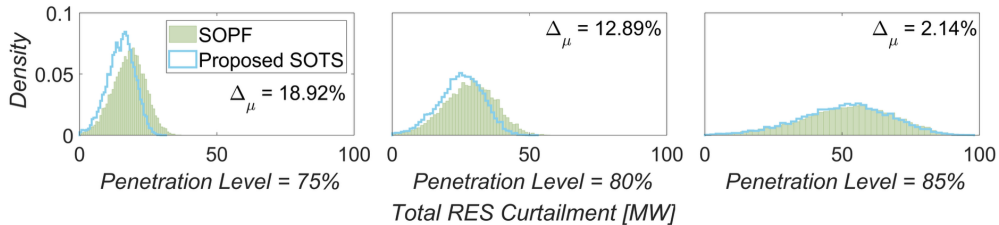


Figure 3: Total RES curtailment PDFs in SOPF and proposed SOTS

The computational comparison in Case 1 explains the practical value of binary relaxation. Figure 4 shows that the MINLP formulation is much slower than the proposed relaxed SOTS. The reported average computation times are 12, 11, 17, and 13 times longer for the MINLP formulation than for the proposed SOTS at penetration levels of 0%, 30%, 60%, and 90%, respectively. The increase in SOTS time at 90% penetration is associated with the computation of curtailment PDFs. At lower penetration levels, the grid does not require renewable curtailment, so the distributional curtailment layer is less demanding.

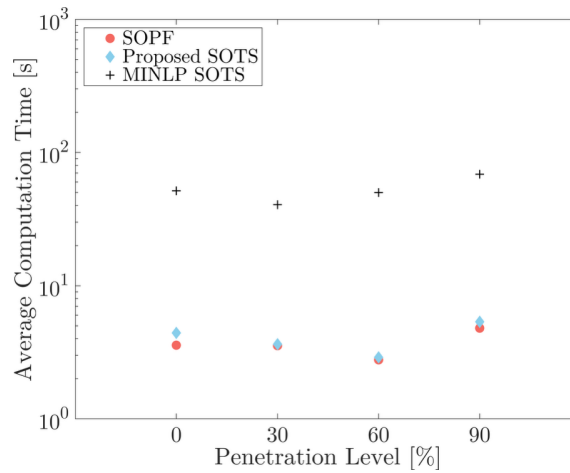


Figure 4: Average computation times of SOPF, MINLP SOTS, and the proposed relaxed SOTS

Case 2 studies the sensitivity of the proposed model on a modified 67-bus hybrid AC/DC test system with 67 AC buses, 17 synchronous generators, 3 wind farms, 102 AC branches, 9 DC buses with converters, and 12 HVDC branches. The proposed model disconnects ten AC branches, whereas the MINLP SOTS does not converge within the 24-hour time limit. The sensitivity test evaluates binary relaxation values of $1e-8$, $1e-6$, $1e-4$, and $1e-2$ with solver tolerances of $1e-6$ and $1e-4$. With solver tolerance $1e-6$, the output distributions align with the SOPF benchmark in which the switched lines are manually removed. With solver tolerance $1e-4$, the binary relaxation value becomes critical. The paper identifies three patterns: variables that fail to align for tolerance $1e-4$, variables that align only when the relaxation parameter equals the solver tolerance, and variables that align for all relaxation values except $1e-8$. The conclusion is operationally useful: choosing an extremely small binary relaxation value is not automatically more accurate if solver tolerance is loose. The relaxation parameter and solver tolerance should be chosen jointly, with solver tolerance no more than two orders of magnitude above the relaxation value.

Case 3 provides the scalability results that must be included in the project final report. The 588-bus hybrid AC/DC system includes 588 AC buses, 167 synchronous generators, 40 wind farms, 686 AC branches, 7 DC buses, 7 converters, and 8 HVDC branches. Table 1 summarizes the reported average computation times and the selected disconnected AC branches. All simulation runs are completed without reaching the l_{opt} iteration limit of 5000. When six or more AC branches can be switched, the model disconnects five AC branches at 30% penetration and four AC branches at 60% and 90% penetration. When the limit is three, the constraint becomes binding and the model disconnects three branches at 30% penetration and two branches at 60% and 90%. No DC branches are disconnected in any setting. The computation times remain between roughly 1392 seconds and 2186 seconds, which confirms tractability for a large stochastic switching problem.

Max switchable AC branches	Penetration	Average time [s]	Disconnected AC branches
24	30%	2109.04	72, 206, 281, 449, 561
24	60%	2163.02	206, 281, 449, 561
24	90%	2186.07	206, 281, 449, 561
12	30%	1839.62	72, 206, 281, 449, 561
12	60%	1925.24	206, 281, 449, 561
12	90%	1885.83	206, 281, 449, 561
6	30%	1392.15	72, 206, 281, 449, 561
6	60%	1434.58	206, 281, 449, 561
6	90%	1431.89	206, 281, 449, 561
3	30%	1675.77	72, 281, 449
3	60%	1459.44	281, 449
3	90%	1726.20	281, 449

4. Expanded interpretation of the complete result set

The paper contains three distinct result layers: economic value on the 5-bus system, numerical sensitivity on the 67-bus system, and scalability on the 588-bus system. The first layer demonstrates why stochastic transmission switching is useful. By disconnecting one AC branch and one DC branch in the 5-bus case, the model changes the feasible power flow pattern in a way that lowers expected cost and reduces curtailment. The savings increase with RES penetration up to about 65%, then continue more slowly until about 80%, and then diminish at very high penetration because the assumed placement of wind generation on all buses reduces the value of additional topology changes. This non-monotonic effect is important because it shows that transmission switching value depends on the spatial and statistical structure of renewable injections, not only on the total penetration level.

The comparison with the MINLP formulation demonstrates that the binary relaxation preserves the economic solution quality. Up to 75% penetration, the cost difference between the relaxed SOTS model and the MINLP SOTS model is effectively zero. Above 75%, the difference remains below 1%, even though the model begins to calculate nonzero curtailment distributions. At the same time, the relaxed formulation is much faster. The MINLP model requires 12, 11, 17, and 13 times more average computation time than the proposed SOTS model at 0%, 30%, 60%, and 90% penetration, respectively. This shows that the relaxation is not merely a mathematical convenience; it makes the switching layer compatible with a stochastic optimization workflow.

The second case study provides a practical warning about the numerical treatment of relaxed binary variables. On the 67-bus hybrid AC/DC system, the proposed model disconnects ten AC branches, whereas the MINLP formulation fails to converge within the 24-hour time limit. The sensitivity analysis shows that the accuracy of output PDFs depends jointly on the binary relaxation parameter and the solver tolerance. With a solver tolerance of $1e-6$, the output PDFs align well across the tested relaxation parameters. With a looser tolerance of $1e-4$, however, the relaxation parameter affects whether the distributions remain consistent. This means that simply choosing an extremely small relaxation parameter does not guarantee better stochastic outputs if the solver is not sufficiently accurate. The practical recommendation is that solver tolerance should be no more than about two orders of magnitude above the binary relaxation parameter.

The scalability study is a central result for the project final report. The 588-bus hybrid AC/DC system contains 588 AC buses, 167 synchronous generators, 40 wind farms, 686 AC branches, 7 DC buses, 7 converters, and 8 HVDC branches. The proposed model is tested under 30%, 60%, and 90% RES penetration and with limits of 24, 12, 6, and 3 switchable AC branches. When the switching limit is not binding, reducing the allowed set of switching candidates lowers computation time because the optimization problem becomes more structured. When the limit becomes binding at three switchable branches, computation time increases again because the model must satisfy a more restrictive combinatorial condition. Importantly, all runs are completed without reaching the 5000-iteration l_{opt} limit, and no DC branch is disconnected in the tested cases.